

Holographic Charged Fluid with Anomalous Current at Finite Cutoff Surface in Einstein-Maxwell Gravity

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The holographic charged fluid with anomalous current in Einstein-Maxwell gravity has been generalized from the infinite boundary to the finite cutoff surface by using the gravity/fluid correspondence. After perturbing the boosted Reissner-Nordstrom (RN)-AdS black brane solution of the Einstein-Maxwell gravity with the Chern-Simons term, we obtain the first order perturbative gravitational and Maxwell solutions, and calculate the stress tensor and charged current of the dual fluid at finite cutoff surfaces. We find that there are nine parameters related to the boundary and gauge conditions one needs to fix before further extracting the information of transport coefficients of the dual fluid. For simplicity, we adopt the Dirichlet boundary condition and Landau gauge to fix these parameters, and obtain the dependence of transport coefficients in the dual stress tensor and charged current on the arbitrary radial cutoff r_c . In addition, we also find that there is no bulk viscosity in this case.

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I. INTRODUCTION

The AdS/CFT correspondence [1–4] provides a remarkable connection between a gravitational theory and a quantum field theory. According to the correspondence, the gravitational theory in an asymptotically AdS spacetime can be formulated in terms of a quantum field theory on its boundary. Particularly, the dynamics of a classical gravitational theory in the bulk is mapped into the strongly coupled quantum field theory on the boundary. Therefore, AdS/CFT provides a useful tool and some insights to investigate the strongly coupled field theory from the dual classical gravitational theory [5, 6].

Usually, the dual field theory in AdS/CFT correspondence resides in the boundary with infinite radial coordinate. Recently, many works have been paid attention to discuss the dual physics at the finite cutoff surface, i.e. the boundary with finite radial coordinate, since it has been shown that a linearized Navier-Stokes (NS) fluid could live on the cutoff surface $r = r_c$, which implies the deep relationship between the NS equations and gravitational equations [7–11]. This deep relationship is further confirmed by the fact that a given solution of the incompressible NS equations could be mapped to a unique solution of the vacuum Einstein equations [8]. In addition, according to the renormalization group (RG) viewpoint, the radial direction of the bulk spacetime corresponds to different energy scale of the dual field theory [12–14]. The energy scale on the infinite boundary is the UV fixed point value, and hence it could not be reached by experiments. Therefore, the physics at finite cutoff surface $r = r_c$ which means finite energy scale becomes important, and the dependence of transport coefficient of dual fluid on the cutoff surface r_c is usually interpreted as the corresponding Wilson renormalization group flow [7, 15–18].

In this paper, we generalize the investigation of the dual fluid on infinite boundary to the finite boundary by using the gravity/fluid correspondence [19]. The gravity/fluid correspondence could be considered as the long wave-length limit of the AdS/CFT correspondence, since the hydrodynamics can be viewed as an effective description of an interacting quantum field theory in the long wave-length limit, i.e. when the length scales under consideration are much larger than the correlation length of the quantum field theory. The big advantage of this correspondence is that it provides a systematic way that maps the boundary fluid to the bulk gravity, since it can construct the stress-energy tensor of the fluid order by order in a derivative expansion from the bulk gravity solution, while the shear viscosity η , entropy density s and the ratio of the shear viscosity over entropy density η/s can all be calculated from the first order stress-energy tensor [20–25]. In addition, besides the stress-energy tensor, this correspondence can also construct the dual

conserved charged current if we introduce the Maxwell field in the bulk gravity, and the information of the thermal conductivity and electrical conductivity of the boundary fluid could be usually extracted from the conserved charged current [20–23]. Furthermore, an interesting case has been found that new effect such as anomalous vortical effect can be brought into the hydrodynamics after adding the Chern-Simons term of Maxwell field in the action [21, 22, 33–35]. The effect of Chern-Simons term is first discussed in three dimensions where the Maxwell theory becomes massive when the Chern-Simons term is included [36]. In addition, the Chern-Simons term can also affect the Holographic Superconductors in 4 dimensions [37] and the instability of black hole in five-dimensional Maxwell theory with the Chern-Simons term [38]. Therefore, we have also investigated the holographic charged fluid at the finite cut-off surface, which contains the Chern-Simons term of the Maxwell field in the bulk. Like the infinite boundary case and the cutoff surface case without charge [39, 40], after perturbing the boosted Reissner-Nordstrom (RN)-AdS black brane solution of the Einstein-Maxwell gravity, we obtain the first order perturbative gravitational and Maxwell solutions, and calculate the stress-energy tensor and charged current of the dual fluid to first order on the finite cutoff surface. We find that they contain nine unknown parameters related to the boundary conditions and gauge conditions, and we explicitly express the dependence of the dual stress tensor and charged current on these parameters. A little different from the case with infinite boundary, here we adopt the Dirichlet boundary condition and Landau frame to fix these parameters. We find that there is no bulk viscosity in the first order dual stress tensor on the finite cutoff surface in our case. In addition, we also obtain the dependence of transport coefficients in the dual stress tensor and charged current on the radial energy scale r_c .

The rest of the paper is organized as follows. In Sec. II, we briefly review some properties of the RN black brane solution. In Sec. III, we construct the perturbative solution to first order. In Sec. IV, we extract the dual stress-energy tensor and the conserved current from this first order perturbation solution, which contain nine parameters. In Sec. V, we consider the Dirichlet boundary condition and Landau frame to fix these parameters, and obtain the dependence of transport coefficients on the radial energy scale r_c . Sec. VI is devoted to the conclusion and discussion.

II. ACTION AND BLACK BRANE SOLUTION OF THE EINSTEIN-MAXWELL GRAVITY WITH CHERN-SIMONS TERM

The action of the 5-dimensional Einstein-Maxwell gravity with Chern-Simons term can be

$$I = \frac{1}{16\pi G_N} \int_{\mathcal{M}} d^5x \sqrt{-g^{(5)}} (R - 2\Lambda) - \frac{1}{4g^2} \int_{\mathcal{M}} d^5x \sqrt{-g^{(5)}} (F^2 + \frac{4\kappa_{cs}}{3} \epsilon^{LABCD} A_L F_{AB} F_{CD}), \quad (2.1)$$

and the equations of motion are

$$\begin{aligned} R_{AB} - \frac{1}{2} R g_{AB} + \Lambda g_{AB} - \frac{1}{2g^2} \left(F_{AC} F_B{}^C - \frac{1}{4} g_{AB} F^2 \right) &= 0, \\ \nabla_B F^B{}_A - \kappa_{cs} \epsilon_{ABCDE} F^{BC} F^{DE} &= 0, \end{aligned} \quad (2.2)$$

where $\Lambda = -6$ and we set $16\pi G_N = 1$ and $l = 1$ for later convenience, and R is the Ricci scalar. In our paper, the black brane solution we are interested in is the 5-dimensional charged RN-AdS black brane solution [26–28]

$$ds^2 = \frac{dr^2}{r^2 f(r)} + r^2 \left(\sum_{i=1}^3 dx_i^2 \right) - r^2 f(r) dt^2, \quad (2.3)$$

where

$$f(r) = 1 - \frac{2M}{r^4} + \frac{Q^2}{r^6}, \quad F = -g \frac{2\sqrt{3}Q}{r^3} dt \wedge dr. \quad (2.4)$$

Note that, the above RN-AdS black brane solution (2.3) is still the solution of (2.2), although the Chern-Simons term affects the Maxwell equation. From (2.3), we can easily find that the outer horizon of the black brane is located at $r = r_+$, where r_+ is the largest root of $f(r) = 0$, and its Hawking temperature and entropy density are

$$T_+ = \frac{(r^2 f(r))'|_{r=r_+}}{4\pi} = \frac{1}{2\pi r_+^3} (4M - \frac{3Q^2}{r_+^2}), \quad (2.5)$$

$$s = \frac{r_+^3}{4G_N}. \quad (2.6)$$

In addition, this black brane solution (2.3) rewritten in the Eddington-Finkelstein coordinate system is

$$\begin{aligned} ds^2 &= -r^2 f(r) dv^2 + 2dvdr + r^2 (dx^2 + dy^2 + dz^2), \\ F &= -g \frac{2\sqrt{3}Q}{r^3} dv \wedge dr. \end{aligned} \quad (2.7)$$

where $v = t + r_*$ with $dr_* = dr/(r^2 f)$. Therefore, the coordinate singularity problem can be avoided in this coordinate system.

Note that, since we will consider the holographic charged fluid at some cutoff hypersurface $r = r_c$ (here r_c is a constant), thus we can first make a coordinate transformation $v \rightarrow v/\sqrt{f(r_c)}$ on the above solution (2.7) before the boost transformation. The motivation of this coordinate transformation is to make the induced metric on the cutoff surface explicitly conformal to flat metric $ds^2 = r_c^2(-dv^2 + dx^2 + dy^2 + dz^2)$. It should be emphasized that the Hawking temperature in this new coordinates system is $T = T_+/\sqrt{f(r_c)}$, and the RN-AdS black brane solution in the new coordinates system is

$$\begin{aligned} ds^2 &= -\frac{r^2 f(r)}{f(r_c)} dv^2 + \frac{2}{\sqrt{f(r_c)}} dv dr + r^2(dx^2 + dy^2 + dz^2), \\ F &= -g \frac{2\sqrt{3}Q}{r^3 \sqrt{f(r_c)}} dv \wedge dr. \end{aligned} \quad (2.8)$$

Therefore, the 5-dimensional boosted RN-AdS black brane solution is given by

$$ds^2 = -\frac{r^2 f(r)}{f(r_c)} (u_\mu dx^\mu)^2 - \frac{2}{\sqrt{f(r_c)}} u_\mu dx^\mu dr + r^2 P_{\mu\nu} dx^\mu dx^\nu, \quad (2.9)$$

$$A = (A_\mu^{ext} + \frac{\sqrt{3}gQ}{r^2 \sqrt{f(r_c)}} u_\mu) dx^\mu, \quad (2.10)$$

with

$$u^v = \frac{1}{\sqrt{1 - \beta_i^2}}, \quad u^i = \frac{\beta_i}{\sqrt{1 - \beta_i^2}}, \quad P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu, \quad (2.11)$$

where velocities β^i , M , Q and A_μ^{ext} are constants, $x^\mu = (v, x_i)$ are the cutoff boundary coordinates, $P_{\mu\nu}$ is the projector onto spatial directions, and the indices in the boundary are raised and lowered with the Minkowski metric $\eta_{\mu\nu} = \text{diag}\{-, +, +, +\}$. Note that, the external field A_μ^{ext} has been added in (2.10), and adding the external field is very important since several research have showed that some transport coefficients like electric conductivity σ_E , anomalous magnetic conductivity σ_B are related to the external field. In addition, the underlying physical reason for the boost transformation is to make the RN-AdS black brane solution have nonzero velocities.

III. THE FIRST ORDER PERTURBATIVE SOLUTION

In this section, we will consider the region between the outer horizon and cutoff surface $r_+ \leq r \leq r_c$, and then perturb the boosted black brane (2.9) to make its velocity and temperature nonuniform. Therefore, we can extract the viscous information of the dual fluid from this perturbative solution via the Gravity/Fluid correspondence. According to the Gravity/Fluid

correspondence, the perturbation is proceeded in the following. Firstly, we promote the above constant parameters β^i , M , Q , A_μ^{ext} to be slowly-varying functions of the cutoff boundary coordinates $x^\mu = (v, x_i)$. Therefore, the metric (2.9) and Maxwell field (2.10) will no longer be the solutions of the equations of motion (2.2), and we need to add extra gravitational and Maxwell fields to make the equations of motion satisfied again. Before discussing these extra fields in detail, we define the following useful tensors

$$W_{IJ} = R_{IJ} + 4g_{IJ} + \frac{1}{2g^2} \left(F_{IK} F^K{}_J + \frac{1}{6} g_{IJ} F^2 \right), \quad (3.1)$$

$$W_A = \nabla_B F^B{}_A - \kappa_{cs} \epsilon_{ABCDE} F^{BC} F^{DE}, \quad (3.2)$$

where we use the convention $\epsilon_{vxyzr} = +\sqrt{-g}$. Obviously, when we take the parameters β^i , M , Q and A_μ^{ext} as functions of x^μ in (2.9), W_{IJ} and W_A will be nonzero and can be proportional to the derivatives of the parameters. Usually, these nonzero terms $-W_{IJ}$ and $-W_A$ are considered as the source terms S_{IJ} and S_A . Therefore, the extra gravitational and Maxwell fields are added into (2.9) and (2.10) such that they can deduce some correction terms to cancel the source terms and make the equations of motion still satisfied. In this work, we only consider the first order perturbative case. For this first order case, the parameters are expanded around $x^\mu = 0$ to the first order

$$\begin{aligned} \beta_i &= \partial_\mu \beta_i|_{x^\mu=0} x^\mu, \quad M = M(0) + \partial_\mu M|_{x^\mu=0} x^\mu, \quad Q = Q(0) + \partial_\mu Q|_{x^\mu=0} x^\mu, \\ A_\mu^{ext} &= A_\mu^{ext}(0) + \partial_\nu A_\mu^{ext}|_{x^\mu=0} x^\nu, \end{aligned} \quad (3.3)$$

where we have assumed $\beta^i(0) = 0$. After inserting the metric (2.9), (2.10) and (3.3) into W_{IJ} and W_A , the nonzero $-W_{IJ}$, $-W_A$ can be considered as the first order source terms $S_{IJ}^{(1)}$ and $S_A^{(1)}$. Therefore, after fixing some gauge (the ‘background field’ gauge in [19], G represents the perturbed metric with corrections)

$$G_{rr} = 0, \quad G_{r\mu} \propto u_\mu, \quad Tr((G^{(0)})^{-1} G^{(1)}) = 0, \quad (3.4)$$

and considering the spatial $SO(3)$ symmetry preserved in the background metric (2.7), the choice for the first order extra gravitational and Maxwell fields around $x^\mu = 0$ can be

$$ds^{(1)2} = \frac{k(r)}{r^2} dv^2 + 2 \frac{h(r)}{\sqrt{f(r_c)}} dv dr + 2 \frac{j_i(r)}{r^2} dv dx^i + r^2 \left(\alpha_{ij} - \frac{2}{3} h(r) \delta_{ij} \right) dx^i dx^j, \quad (3.5)$$

$$A^{(1)} = a_v(r) dv + a_i(r) dx^i. \quad (3.6)$$

Note that, for gauge field part, $a_r(r)$ does not contribute to field strength, thus the choice $a_r(r) = 0$ is trivial. Therefore, the first order perturbation solution can be obtained from the vanishing

$W_{IJ} = (\text{effect from correction}) - S_{IJ}^{(1)}$ and $W_A = (\text{effect from correction}) - S_A^{(1)}$. Here, the “effect from correction” means the correction to W_{IJ} and W_A from (3.5) and (3.6).

For the first order gravitational equations, they are complicated which have been listed in the appendix A. For the first order Maxwell equations, they are

$$\begin{aligned} W_v &= \frac{f(r)}{r} \left\{ r^3 a_v'(r) - \frac{4\sqrt{3}gQ}{\sqrt{f(r_c)}} h(r) \right\}' - S_v^{(1)}(r) = 0, \\ W_r &= -\frac{1}{r^3} \left\{ r^3 \sqrt{f(r_c)} a_v'(r) - 4\sqrt{3}gQ h(r) \right\}' - S_r^{(1)}(r) = 0, \\ W_i &= \frac{1}{r} \left\{ r^3 f(r) a_i'(r) + \frac{2\sqrt{3}gQ \sqrt{f(r_c)}}{r^4} j_i(r) \right\}' - S_i^{(1)}(r) = 0. \end{aligned} \quad (3.7)$$

where

$$\begin{aligned} S_v^{(1)}(r) &= -g \frac{2\sqrt{3}}{r^3} (\partial_v Q + Q \partial_i \beta_i), \\ S_r^{(1)}(r) &= 0, \\ S_i^{(1)}(r) &= -\frac{F_{vi}^{\text{ext}} \sqrt{f(r_c)}}{r} + g \left(\frac{\sqrt{3}Q \partial_v \beta_i}{r^3} + \frac{\sqrt{3}Q \partial_i M}{r^3 r_c^4 f(r_c)} + \frac{\sqrt{3} \partial_x Q (-2M + r_c^4)}{r^3 r_c^4 f(r_c)} \right) \\ &\quad - \kappa_{\text{cs}} \left(\frac{8\sqrt{3}gQ}{r^4} \epsilon^{ijk} F_{jk}^{\text{ext}} + \frac{48g^2 Q^2}{r^6 \sqrt{f(r_c)}} \epsilon^{ijk} \partial_j \beta_k \right). \end{aligned} \quad (3.8)$$

and $F_{\mu\nu}^{\text{ext}} \equiv \partial_\mu A_\nu^{\text{ext}} - \partial_\nu A_\mu^{\text{ext}}$ is the external field strength tensor, prime means derivative with respect to r coordinate. Note that, the cutoff effect has been implicated in these equations through their dependence on r_c . In addition, from the above first order gravitational and Maxwell equations, there are several interesting relations between these equations

$$\begin{aligned} W_v + \frac{r^2 f(r)}{\sqrt{f(r_c)}} W_r &= 0 : S_v^{(1)} + \frac{r^2 f(r)}{\sqrt{f(r_c)}} S_r^{(1)} = 0, \\ W_{vv} + \frac{r^2 f(r)}{\sqrt{f(r_c)}} W_{vr} &= 0 : S_{vv}^{(1)} + \frac{r^2 f(r)}{\sqrt{f(r_c)}} S_{vr}^{(1)} = 0, \\ W_{vi} + \frac{r^2 f(r)}{\sqrt{f(r_c)}} W_{ri} &= 0 : S_{vi}^{(1)} + \frac{r^2 f(r)}{\sqrt{f(r_c)}} S_{ri}^{(1)} = 0. \end{aligned} \quad (3.9)$$

which can be considered as the constraint equations. In our paper, after using the first order source terms, we can further rewrite these constrain equations (3.9) as

$$\begin{aligned} \partial_v Q + Q \partial_i \beta_i &= 0, \\ 3\partial_v M + 4M \partial_i \beta_i &= 0, \\ \partial_i M + 4M \partial_v \beta_i &= -\sqrt{f(r_c)} \frac{\sqrt{3}Q}{g} F_{vi}^{\text{ext}} + \frac{4M (Q \partial_i Q - r_c^2 \partial_i M)}{f(r_c) r_c^6}. \end{aligned} \quad (3.10)$$

Later, we can see that there are some underlying physical interpretations for these equations, i.e., these equations are related to the conservation equations of the zeroth order stress-energy tensor and conserved current.

After solving the above equations, several coefficients of the first order extra gravitational and Maxwell fields are

$$\begin{aligned}
h(r) &= C_{h2} + \frac{C_{h1}}{r^4} \\
a_v(r) &= C_{av2} + \frac{C_{av1}}{r^2} - \frac{2C_{h1}gQ}{\sqrt{3}r^6\sqrt{f(r_c)}} \\
k(r) &= C_{k2} + C_{k1}r^2 - \frac{2C_{h2}r^4}{f(r_c)} + \frac{4C_{h1}(-Q^2 + Mr^2)}{3r^6f(r_c)} + \frac{2C_{av1}Q}{\sqrt{3}gr^2\sqrt{f(r_c)}} + \frac{2r^3\partial_i\beta_i}{3\sqrt{f(r_c)}} \\
\alpha_{ij}(r) &= \alpha(r) \left\{ (\partial_i\beta_j + \partial_j\beta_i) - \frac{2}{3}\delta_{ij}\partial_k\beta^k \right\},
\end{aligned} \tag{3.11}$$

where $\alpha(r)$ is

$$\alpha(r) = \int_{r_c}^r \frac{s^3 - r_+^3}{-s^5 f(s)} ds. \tag{3.12}$$

Note that, $h(r)$ is solved from $W_{rr} = 0$, while $a_v(r)$ is solved from $W_r = 0$ or $W_v = 0$, and $k(r)$ is solved from $W_{vv} = 0$ or $W_{vr} = 0$. For the exact solutions to $j_i(r)$ and $a_i(r)$, it is more difficult to solve them since $j_i(r)$ and $a_i(r)$ couple with each other. The detailed solving the corresponding differential equations can be seen in appendix B.

Therefore, after adding the correction terms, the first-order metric expanded in boundary derivatives around $x^\mu = 0$ can be explicitly given as

$$\begin{aligned}
ds^2 &= \frac{2}{\sqrt{f(r_c)}} dv dr - \frac{r^2}{f(r_c)} f(M_0, Q_0, r) dv^2 + r^2 dx_i^2 - \frac{r^2}{f(r_c)} x^\mu C_1(r) \partial_\mu M dv^2 - 2x^\mu \partial_\mu \beta_i dx^i dr \\
&\quad - \frac{2}{\sqrt{f(r_c)}} x^\mu r^2 [1 - f(M_0, Q_0, r)] \partial_\mu \beta_i dx^i dv - \frac{r^2}{f(r_c)} x^\mu C_2(r) \partial_\mu Q dv^2 + \frac{k(r)}{r^2} dv^2 \\
&\quad + 2 \frac{h(r)}{\sqrt{f(r_c)}} dv dr + 2 \frac{j_i(r)}{r^2} dv dx^i + 2r^2 \left(\sigma_{ij} - \frac{1}{3} h(r) \delta_{ij} \right) dx^i dx^j,
\end{aligned} \tag{3.13}$$

where

$$\begin{aligned}
f(M_0, Q_0, r) &= f(M(x^\mu), Q(x^\mu), r)|_{x^\mu=0}, \quad C_1(r) = \frac{\partial f(M(x^\mu), Q(x^\mu), r)}{\partial M}|_{x^\mu=0}, \\
C_2(r) &= \frac{\partial f(M(x^\mu), Q(x^\mu), r)}{\partial Q}|_{x^\mu=0}, \quad \sigma_{ij} = \partial_{(i}\beta_{j)} - \frac{1}{3}\delta_{ij}\partial_k\beta^k.
\end{aligned} \tag{3.14}$$

and the global first-order metric defined on the whole cutoff surface can be constructed by replacing (3.13) in a covariant form, i.e., replacing $\partial_i\beta^i$ as $\partial_\lambda u^\lambda$ in (3.13) [19].

IV. THE STRESS TENSOR AND CHARGED CURRENT OF DUAL FLUID ON THE CUTOFF SURFACE VIA GRAVITY/FLUID CORRESPONDENCE

The information of the dual fluid like its stress tensor and charged current can be directly extracted from the above global first-order perturbative solution. In addition, we can also first extract the dual stress tensor and charged current from (3.13), and then rewrite the dual stress tensor and charged current in a covariant form. Here we use the later calculation.

According to the Gravity/Fluid correspondence, the dual stress tensor $\tau_{\mu\nu}$ can be obtained through the following relation [29]

$$\sqrt{-h}h^{\mu\nu}\langle\tau_{\nu\sigma}\rangle = \lim_{r\rightarrow r_c} \sqrt{-\gamma}\gamma^{\mu\nu}T_{\nu\sigma}. \quad (4.1)$$

where $h_{\mu\nu}$ is the background metric upon which the dual field theory resides, $\gamma^{\mu\nu}$ is the boundary metric on the cutoff surface obtained from the well-known ADM decomposition

$$ds^2 = \gamma_{\mu\nu}(dx^\mu + V^\mu dr)(dx^\nu + V^\nu dr) + N^2 dr^2, \quad (4.2)$$

and $T_{\mu\nu}$ is the corresponding boundary stress tensor which is [30–32]

$$T_{\mu\nu} = 2(K_{\mu\nu} - K\gamma_{\mu\nu} - C\gamma_{\mu\nu}), \quad (4.3)$$

where the extrinsic curvature is $K_{\mu\nu} = -\frac{1}{2}(\nabla_\mu n_\nu + \nabla_\nu n_\mu)$ and n^μ is the normal vector of the constant hypersurface $r = r_c$ pointing toward the r increasing direction. Note that, the term $C\gamma_{\mu\nu}$ in (4.3) relates to the boundary counterterm which is usually added to cancel the divergence of the boundary stress tensor when the boundary $r = r_c$ approaches to infinity, i.e., $C = 3$ in the asymptotical AdS_5 case. However, there is no divergence of the boundary stress tensor in our case with finite boundary, here the reason that we still add the boundary counterterm is to compare with the previous results when r_c approaches to infinity.

Note that, the dual fluid usually resides on the flat spacetime $\eta_{\mu\nu}$, i.e., $h_{\mu\nu} = \eta_{\mu\nu}$. Therefore, the boundary induced metric $\gamma_{\mu\nu}$ is also usually chosen to be conformal to $\eta_{\mu\nu}$. In our case, we can consider the simple case to make $\gamma_{\mu\nu}$ conformal to $\eta_{\mu\nu}$. That is, we can impose the Dirichlet boundary condition

$$h(r_c) = 0, \quad k(r_c) = 0, \quad j_i(r_c) = 0, \quad a_v(r_c) = 0, \quad a_i(r_c) = 0. \quad (4.4)$$

Therefore, we can obtain that the induced metric $\gamma_{\mu\nu} = r_c^2 \eta_{\mu\nu}$. According to (4.1), the expectation value of the first order stress tensor of the dual fluid $\tau_{\mu\nu}$ is

$$\tau_{\mu\nu} = r_c^2 T_{\mu\nu}. \quad (4.5)$$

where the boundary stress tensor $T_{\mu\nu}$ could be explicitly expressed from (3.13) by using (4.3)

$$\begin{aligned}
T_{vv}^{(0)} &= 2 \left(C - 3\sqrt{f(r_c)} \right) r_c^2 \\
T_{ii}^{(0)} &= \frac{-4M + 2 \left(3 - C\sqrt{f(r_c)} \right) r_c^4}{\sqrt{f(r_c)} r_c^2} \\
T_{vv}^{(1)} &= -2\partial_i \beta_i r_c + 6\sqrt{f(r_c)} r_c^2 h(r_c) + \frac{\left(-2C + 9\sqrt{f(r_c)} \right) k(r_c)}{r_c^2} + 2\sqrt{f(r_c)} r_c^3 h'(r_c) \\
T_{vi}^{(1)} &= -\frac{Q\partial_i Q}{f(r_c) r_c^5} + \frac{\partial_i M}{f(r_c) r_c^3} - \partial_v \beta_i r_c \\
&\quad + 2 \frac{\left(-Q^2 + \left(2 - C\sqrt{f(r_c)} + 3f(r_c) \right) r_c^6 \right) j_i(r_c)}{\sqrt{f(r_c)} r_c^8} - \frac{\sqrt{f(r_c)} j'_i(r_c)}{r_c} \\
T_{ij}^{(1)} &= 2 \left(2\delta_{ij} \partial_k \beta_k - \partial_{(i} \beta_{j)} \right) r_c + 2\delta_{ij} \frac{\partial_k \beta_k (2M - 3r_c^4)}{3f(r_c) r_c^3} \\
&\quad + 2 \left(-Cr_c^2 + \frac{-2M + 3r_c^4}{\sqrt{f(r_c)} r_c^2} \right) a_{ij}(r_c) - \sqrt{f(r_c)} r_c^3 a'_{ij}(r_c) \\
&\quad + 2\delta_{ij} \left(\left(\frac{2cr_c^2}{3} + \frac{5(2M - 3r_c^4)}{3\sqrt{f(r_c)} r_c^2} \right) h(r_c) - \frac{2}{3} \sqrt{f(r_c)} r_c^3 h'(r_c) \right) \\
&\quad + 2\delta_{ij} \left(\frac{(-2M + (3 - 2f(r_c)) r_c^4) k(r_c)}{2\sqrt{f(r_c)} r_c^6} - \frac{\sqrt{f(r_c)} k'(r_c)}{2r_c} \right)
\end{aligned} \tag{4.6}$$

In addition, for the dual charged current on the cutoff surface, it can be computed via

$$J^\mu = \lim_{r \rightarrow r_c} r_c^4 \frac{1}{\sqrt{-\gamma}} \frac{\delta S_{cl}}{\delta \tilde{A}_\mu} = \lim_{r \rightarrow r_c} r_c^4 \frac{N}{g^2} (F^{r\mu} + \frac{4\kappa_{cs}}{3} \epsilon^{r\mu\rho\sigma\tau} A_\rho F_{\sigma\tau}) , \tag{4.7}$$

where \tilde{A}_μ is the gauge field projected to boundary, and the extra term from the Chern-Simons term has been added.

$$J_{(1)}^\nu = \frac{2\sqrt{3}Qh(r_c)}{g} - \frac{\sqrt{3}Qk(r_c)}{gr_c^4} - \frac{\sqrt{f(r_c)} r_c^3 a'_v(r_c)}{g^2}, \tag{4.8}$$

$$\begin{aligned}
J_{(1)}^i &= \frac{2\sqrt{3}Qj_i(r_+)}{gr_+^4} + \frac{r_+ F_{vi}^{\text{ext}}}{g^2} + \frac{\sqrt{3}Q\partial_v \beta_i}{gr_+ \sqrt{f(r_c)}} + \frac{\sqrt{3}Q\partial_i M}{gr_+ r_c^4 f(r_c)^{3/2}} + \frac{\sqrt{3}\partial_i Q (-Q^2 + r_c^6 f(r_c))}{gr_+ r_c^6 f(r_c)^{3/2}} \\
&\quad + \kappa_{cs} \left(\frac{4Q^2 (r_+^4 - 3r_c^4)}{r_+^4 r_c^4 f(r_c)} \epsilon^{ijk} \partial_j \beta_k + \frac{4Q (2r_+^2 - 3r_c^2)}{\sqrt{3}gr_+^2 r_c^2 \sqrt{f(r_c)}} \epsilon^{ijk} F_{jk}^{\text{ext}} \right),
\end{aligned} \tag{4.9}$$

where $j_x(r_+)/r_+^4$ is a little complicate, and it has been listed in the (B15) in appendix B. In addition, we have set $A_\mu^{\text{ext}}(0) = 0$ for the simplicity. Therefore, we have obtained the stress tensor and charged current of dual fluid according to the gravity/fluid correspondence. In the following, we will further fix the parameters to explicitly extract the information of transport coefficients in the fluid, where more conditions should be considered.

V. FIXING THE PARAMETERS BY USING THE SUITABLE CONDITIONS

Note that, there are nine parameters $C_{h1}, C_{h2}, C_{k1}, C_{k2}, C_{av1}, C_{av2}, C_3, C_4$ and C contained in the expressions of stress tensor and charged current of the dual fluid. In order to fix these parameters, we should use several suitable conditions. First, we note that C_{k1} should vanish for the solution of eq. (A6) and (A8) to be consistent, which could be checked by inserting the expressions $h(r), a_v(r), \alpha_{ii}(r)$ into $W_{ii} = 0$ in (A6). Second, we can choose the Landau frame to obtain the well-defined energy density and pressure, which means the following three components must vanish

$$\begin{aligned}
\tau_{vv}^{(1)} &= -\frac{4\partial_i\beta_i r_c (C - 3\sqrt{f(r_c)})}{3\sqrt{f(r_c)}} + C_{k1} (-2C + 9\sqrt{f(r_c)}) + \frac{C_{k2} (-2C + 9\sqrt{f(r_c)})}{r_c^2} \\
&\quad + \frac{2C_{h1} (2C (Q^2 + r_c^6 (-1 + f(r_c))) - 3\sqrt{f(r_c)} (3Q^2 + r_c^6 (-3 + 4f(r_c))))}{3r_c^8 f(r_c)} \\
&\quad + \frac{2C_{h2} r_c^2 (2C + 3(-3 + f(r_c)) \sqrt{f(r_c)})}{f(r_c)} + \frac{2C_{av1} Q (-2C + 9\sqrt{f(r_c)})}{\sqrt{3}gr_c^4 \sqrt{f(r_c)}} \\
\tau_{vx}^{(1)} &= \frac{C_4}{\sqrt{f(r_c)}} + 2C_3 r_c^4 (-C + 3\sqrt{f(r_c)}) f(r_c) \\
&\quad + \frac{-Q\partial_x Q + r_c^2 (\partial_x M - \partial_v \beta_x r_c^4 (-\sqrt{f(r_c)} + f(r_c)))}{r_c^3 f(r_c)} \\
J_{(1)}^v &= -\frac{2Q\partial_i\beta_i}{\sqrt{3}gr_c \sqrt{f(r_c)}} - \frac{\sqrt{3}C_{k1}Q}{gr_c^2} - \frac{\sqrt{3}C_{k2}Q}{gr_c^4} - \frac{2C_{h1}Q (-2Q^2 + 2Mr_c^2 + 3r_c^6 f(r_c))}{\sqrt{3}gr_c^{10} f(r_c)} \\
&\quad + \frac{2\sqrt{3}C_{h2}Q (1 + f(r_c))}{gf(r_c)} + \frac{2C_{av1} (-Q^2 + r_c^6 f(r_c))}{g^2 r_c^6 \sqrt{f(r_c)}}. \tag{5.1}
\end{aligned}$$

Third, we can choose $C = 3$. Therefore, it is convenient to compare our results with the previous results when r_c approaches infinite.

After using these three suitable conditions and the Dirichlet boundary condition in the above section, we can find that the nine parameters can be fixed

$$\begin{aligned}
C_{h1} &= -\frac{\partial_i\beta_i r_c^3}{4\sqrt{f(r_c)}}, \quad C_{h2} = \frac{\partial_i\beta_i}{4\sqrt{f(r_c)}r_c}, \quad C_{k1} = 0, \\
C_{av1} &= -\frac{\sqrt{3}gQ\partial_i\beta_i}{2f(r_c)r_c}, \quad C_{av2} = \frac{gQ\partial_i\beta_i}{\sqrt{3}f(r_c)r_c^3}, \quad C_{k2} = -\frac{\partial_i\beta_i (-10M + r_c^4)}{6f(r_c)^{3/2}r_c}, \\
C_3 &= 0, \quad C_4 = \partial_v \beta_x r_c^3 (-1 + \sqrt{f(r_c)}) + \frac{Q\partial_x Q - \partial_x M r_c^2}{r_c^3 \sqrt{f(r_c)}}, \quad C = 3. \tag{5.2}
\end{aligned}$$

Therefore, the non-zero components of boundary fluid energy stress tensor are

$$\begin{aligned}
\tau_{vv}^{(0)} &= 6 (1 - \sqrt{f(r_c)}) r_c^4, \quad \tau_{ii}^{(0)} = \frac{-4M + 6 (1 - \sqrt{f(r_c)}) r_c^4}{\sqrt{f(r_c)}}, \\
\tau_{ij}^{(1)} &= -2r_+^3 \sigma_{ij}, \tag{5.3}
\end{aligned}$$

which can be further rewritten in a covariant form

$$\tau_{\mu\nu} = (\rho + p)u_\mu u_\nu + p\eta_{\mu\nu} - 2\eta\sigma_{\mu\nu}, \quad (5.4)$$

From (5.4), we can read out the energy density ρ , pressure p and shear viscosity η of the dual fluid on the cutoff surface

$$\rho = 6 \left(1 - \sqrt{f(r_c)}\right) r_c^4, \quad p = \frac{-4M + 6 \left(1 - \sqrt{f(r_c)}\right) r_c^4}{\sqrt{f(r_c)}}, \quad \eta = r_+^3. \quad (5.5)$$

The bulk viscosity is absent in this case.

The entropy density s of dual fluid can be computed through

$$s = \left(\frac{\partial p}{\partial T}\right)_\mu = 4\pi r_+^3. \quad (5.6)$$

which is consistent with the entropy density of the black brane solution in (2.6). It is convenient to check this equation (5.6) if we express p in the functions of r_+ and Q . The shear viscosity to entropy ratio is universally $1/4\pi$.

The zeroth and first order charged current of the dual fluid are

$$J_{(0)}^\mu = -\frac{2\sqrt{3}Q}{g}u^\mu =: nu^\mu, \quad (5.7)$$

$$J_{(1)}^\mu = -\kappa P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma_E E^\mu + \sigma_B B^\mu + \xi\omega^\mu, \quad (5.8)$$

where n is particle number density and

$$E^\mu = u^\lambda \tilde{F}_\lambda^\mu, \quad B^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu \tilde{F}_{\rho\sigma}, \quad \omega^\mu = \epsilon^{\mu\nu\rho\sigma}u_\nu \partial_\rho u_\sigma. \quad (5.9)$$

Note that, here $\tilde{F}_{\mu\nu}$ is defined at the cutoff surface $r = r_c$ through $A_\mu = A_\mu^{\text{ext}} + \frac{\sqrt{3}gQ}{r_c^2\sqrt{f(r_c)}}u_\mu$.

Therefore, electric and magnetic fields are given by

$$\begin{aligned} E^i &= F_{vi}^{\text{ext}} \left(1 - \frac{3Q^2}{4Mr_c^2}\right) - \frac{\sqrt{3}gQ\partial_i M}{4Mr_c^2\sqrt{f(r_c)}} + \frac{\sqrt{3}g\partial_i Q}{r_c^2\sqrt{f(r_c)}}, \\ B^i &= -\frac{1}{2}\epsilon^{ijk} \left(F_{jk}^{\text{ext}} + \frac{2\sqrt{3}gQ\partial_j\beta_k}{r_c^2\sqrt{f(r_c)}}\right), \quad \omega^i = -\epsilon^{ijk}\partial_j\beta_k. \end{aligned} \quad (5.10)$$

The chemical potential is defined as

$$\mu = A_v(r_+) - A_v(r_c). \quad (5.11)$$

Using the same discussion in reference [20], we can find that its first order expression is

$$\mu = \frac{\sqrt{3}gQ}{\sqrt{f(r_c)}} \left(\frac{1}{r_c^2} - \frac{1}{r_+^2}\right). \quad (5.12)$$

which keeps the same expression but here M , Q and r_+ are not constants.

Furthermore, the familiar thermodynamic relation still holds on arbitrary cutoff surface

$$\rho + p - sT = n\mu, \quad (5.13)$$

where T is the temperature of the dual fluid given by the Hawking temperature of the black brane solution $T = \frac{T_+}{\sqrt{f(r_c)}}$, n is particle number density defined in (5.7). Thus we can conclude that the thermodynamical properties of the dual charged fluid are universal on the cutoff surface.

The transport coefficients are found to be

$$\begin{aligned} \kappa &= \frac{16\pi^2 r_+^7 T_+^3}{g^2 r_c^{10} \sqrt{f(r_c)} f'(r_c)^2}, \quad \sigma_E = \frac{16\pi^2 r_+^7 T_+^2}{g^2 r_c^{10} f'(r_c)^2}, \\ \sigma_B &= -\frac{8Q(3r_c^2 - 2r_+^2) \kappa_{cs}}{\sqrt{3} g r_+^2 r_c^2 \sqrt{f(r_c)}} + \frac{24\sqrt{3} Q^3 (r_c^2 - r_+^2)^2 \kappa_{cs}}{g r_+^4 r_c^9 \sqrt{f(r_c)} f'(r_c)}, \\ \xi &= \frac{12Q^2 (r_c^2 - r_+^2)^2 \kappa_{cs}}{r_+^4 r_c^4 f(r_c)} - \frac{48Q^4 (r_c^2 - r_+^2)^3 \kappa_{cs}}{r_+^6 r_c^{11} f(r_c) f'(r_c)}. \end{aligned} \quad (5.14)$$

In the large r_c limit, they can be expanded in the power series of r_c

$$\begin{aligned} \kappa &= \frac{\pi^2 r_+^7 T_+^3}{4g^2 M^2} + \frac{3\pi^2 Q^2 r_+^7 T_+^3}{8g^2 M^3 r_c^2} + \mathcal{O}(r_c^{-4}), \\ \sigma_E &= \frac{\pi^2 r_+^7 T_+^2}{4g^2 M^2} + \frac{3\pi^2 Q^2 r_+^7 T_+^2}{8g^2 M^3 r_c^2} + \mathcal{O}(r_c^{-4}), \\ \sigma_B &= -\frac{\sqrt{3} Q (2M + 3r_+^4) \kappa_{cs}}{g M r_+^2} + \frac{Q (28M^2 - 36M r_+^4 + 27r_+^8) \kappa_{cs}}{4\sqrt{3} g M^2 r_c^2} + \mathcal{O}(r_c^{-4}), \\ \xi &= \frac{6Q^2 \kappa_{cs}}{M} - \frac{3Q^2 (4M^2 + 3r_+^8) \kappa_{cs}}{2M^2 r_+^2 r_c^2} + \mathcal{O}(r_c^{-4}). \end{aligned} \quad (5.15)$$

From which, it can be easily found that our results can return to the previous results where the dual fluid is on the infinity boundary, i.e., the zero order coefficients in (5.15) are same as the previous results in [25].

We find that the thermal conductivity and electrical conductivity satisfy the so-called Wiedermann-Franz law at arbitrary cutoff surface [20, 25]

$$\kappa = \sigma_E T. \quad (5.16)$$

The anomaly coefficients σ_B and ξ can be further written in following form

$$\sigma_B = c \left(\mu - \frac{1}{2} \frac{n\mu^2}{\rho + p} \right) - \frac{c Q}{r_c^2 \sqrt{3f(r_c)}}, \quad (5.17)$$

$$\xi = \frac{c}{2} \left(\mu^2 - \frac{2}{3} \frac{n\mu^3}{\rho + p} \right). \quad (5.18)$$

where anomalous factor $c = 8\kappa_{cs}/g^2$. The second term in eq. (5.17) comes from terms proportional to $\kappa_{cs}\epsilon^{\mu\rho\sigma\tau}A_\rho F_{\sigma\tau}$ in the dual current (4.7), which vanishes asymptotically. Taking into account a factor 1/2 in the definition of ω^μ , we can recover the result in [33, 41]. Thus the Chern-Simons term would contribute the anomalous coefficients a non-trivial evolution with the cutoff surface.

In addition, after a little work, we can find that the constraint equation (3.9) can be expressed covariantly as

$$\begin{aligned}\partial_\mu J_{(0)}^\mu &= 0, \\ \partial_\mu \tau_{(0)}^{\mu\nu} &= \tilde{F}^{\mu\nu} J_{(0)}^\mu.\end{aligned}\tag{5.19}$$

which are just the exact zeroth order conservation equations.

VI. CONCLUSION

In this paper, we generalize the dual charged fluid on the infinite boundary case to the finite cutoff surface case via the gravity/fluid correspondence. Like the same procedure as the infinite boundary case, we first lift the parameters of the boosted RN black brane in the Einstein-Maxwell gravity with Chern-Simons term to functions of finite cutoff boundary coordinates, and then solving for the corresponding correction terms, we finally construct the first order perturbative gravitational and Maxwell solutions. Basing the perturbative solutions, we extract the information of its dual fluid on the finite cutoff surface such as its stress tensor and charged current. Note that, the stress tensor and charged current on the finite cutoff surface could depend on nine parameters which relate to the boundary and gauge conditions. We explicitly express the dependence of the dual stress tensor and charged current on these parameters. Since different choices of these parameters could corresponds different dual physics on the finite cutoff surface, we mainly use the Dirichlet boundary condition and Landau frame to fix these parameters. We find that there is no bulk viscosity for our case and thermodynamical properties are universal on arbitrary cutoff surface. We also work out the explicit dependence of transport coefficients in the dual stress tensor and charged current on the radial energy scale r_c .

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Appendix A: The tensor components of $W_{\mu\nu}$ and $S_{\mu\nu}$

The tensor components of $W_{\mu\nu} = (\text{effect from correction}) - S_{\mu\nu}$ are

$$W_{vv} = -\frac{8r^2 f(r) h(r)}{f(r_c)} + \frac{2(2Q^2 - 2Mr^2 - r^6) f(r) h'(r)}{r^3 f(r_c)} - \frac{4Q f(r) a'_v(r)}{\sqrt{3}gr\sqrt{f(r_c)}} \quad (\text{A1})$$

$$+ \frac{f(r) k'(r)}{2r} - \frac{1}{2} f(r) k''(r) - S_{vv}^{(1)},$$

$$W_{vi} = -\frac{\sqrt{3}Q f(r) a'_x(r)}{gr\sqrt{f(r_c)}} + \frac{3f(r) j'_x(r)}{2r} - \frac{1}{2} f(r) j''_x(r) - S_{vi}^{(1)}(r), \quad (\text{A2})$$

$$W_{vr} = \frac{4Q a'_v(r)}{\sqrt{3}gr^3} + \frac{8h(r)}{\sqrt{f(r_c)}} + \frac{2(-2Q^2 + 2Mr^2 + r^6) h'(r)}{r^5 \sqrt{f(r_c)}} \\ - \frac{\sqrt{f(r_c)} k'(r)}{2r^3} + \frac{\sqrt{f(r_c)} k''(r)}{2r^2} - S_{vr}^{(1)} \quad (\text{A3})$$

$$W_{ri} = \frac{\sqrt{3}Q a'_i(r)}{gr^3} - \frac{3\sqrt{f(r_c)} j'_i(r)}{2r^3} + \frac{\sqrt{f(r_c)} j''_i(r)}{2r^2} - S_{ri}^{(1)} \quad (\text{A4})$$

$$W_{rr} = \frac{5h'(r)}{r} + h''(r) - S_{rr}^{(1)} \quad (\text{A5})$$

$$W_{ii} = 8r^2 h(r) + \frac{(5Q^2 - 14Mr^2 + 11r^6) h'(r)}{3r^3} + \frac{1}{3} r^4 f(r) h''(r) + \frac{f(r_c) k'(r)}{r} \\ - \frac{2Q \sqrt{f(r_c)} a'_v(r)}{\sqrt{3}gr} + \frac{(Q^2 + 2Mr^2 - 5r^6) \alpha'_{ii}(r)}{2r^3} - \frac{1}{2} r^4 f(r) \alpha''_{ii}(r) - S_{ii}^{(1)} \quad (\text{A6})$$

$$W_{ij} = \frac{(Q^2 + 2Mr^2 - 5r^6) \alpha'_{ij}(r)}{2r^3} - \frac{1}{2} r^4 f(r) \alpha''_{ij}(r) - S_{ij}^{(1)}, \quad (i \neq j) \quad (\text{A7})$$

$$W_{ij} - \frac{1}{3} \delta_{ij} \left(\sum_k W_{kk} \right) = \frac{(Q^2 + 2Mr^2 - 5r^6) \alpha'_{ij}(r)}{2r^3} - \frac{1}{2} r^4 f(r) \alpha''_{ij}(r) - S_{ij}^{(1)} \quad (\text{A8})$$

where the first order source terms are

$$S_{vv}^{(1)}(r) = -\frac{3\partial_v M}{r^3 \sqrt{f(r_c)}} + \frac{3Q\partial_v Q}{r^5 \sqrt{f(r_c)}} - \frac{(-2Q^2 + 2Mr^2 + r^6) \partial_i \beta_i}{r^5 \sqrt{f(r_c)}} \quad (\text{A9})$$

$$S_{vi}^{(1)}(r) = \frac{\sqrt{3}Q F_{vi}^{\text{ext}}}{gr^3} - \frac{Q(3Q^2 + 2Mr^2 + 3r^6) \partial_i Q}{2r^5 r_c^6 f(r_c)^{3/2}} + \frac{(3Q^2 + 2Mr^2 + 3r^6) \partial_i M}{2r^5 r_c^4 f(r_c)^{3/2}} \\ + \frac{\partial_i M}{r^3 \sqrt{f(r_c)}} + \frac{(3Q^2 + 2Mr^2 + 3r^6) \partial_v \beta_i}{2r^5 \sqrt{f(r_c)}} \quad (\text{A10})$$

$$S_{vr}^{(1)}(r) = \frac{\partial_i \beta_i}{r} \quad (\text{A11})$$

$$S_{ri}^{(1)}(r) = -\frac{3\partial_v \beta_i}{2r} + \frac{3Q\partial_i Q}{2r r_c^6 f(r_c)} - \frac{3\partial_i M}{2r r_c^4 f(r_c)} \quad (\text{A12})$$

$$S_{rr}^{(1)}(r) = 0, \quad (\text{A13})$$

$$S_{ij}^{(1)}(r) = (\delta_{ij} \partial_k \beta_k + 3\partial_{(i} \beta_{j)}) r \sqrt{f(r_c)}. \quad (\text{A14})$$

Appendix B: The exact form of $j_i(r)$ and $a_i(r)$

For the $a_i(r)$ and $j_i(r)$, we can solve them from equations $W_i = 0$ and $W_{ri} = 0$. However, these equations are more difficult to solve since $a_i(r)$ and $j_i(r)$ are coupled to each other. These equations $W_i = 0$ and $W_{ri} = 0$ explicitly are

$$\left(r^3 f(r) a_i'(r) + \frac{2\sqrt{3}gQ}{r^4} \sqrt{f(r_c)} j_i(r) \right)' = r S_i^{(1)}(r), \quad (\text{B1})$$

$$\frac{\sqrt{3}Q a_i'(r)}{gr^3} - \frac{3\sqrt{f(r_c)} j_i'(r)}{2r^3} + \frac{\sqrt{f(r_c)} j_i''(r)}{2r^2} = S_{ri}^{(1)}(r). \quad (\text{B2})$$

Since we just consider the first order case, we ignore the up indexes (1) in $S_{ri}^{(1)}(r)$ and $S_i^{(1)}(r)$ for the convenience in the following.

Integrating eq. (B1) from the horizon r_+ to cutoff r_c , we get

$$r^3 f(r) a_i'(r) + 2\sqrt{3}gQ \left(\frac{j_i(r)}{r^4} - \frac{j_i(r_+)}{r_+^4} \right) = \int_{r_+}^r ds s S_i(s), \quad (\text{B3})$$

and imposing the boundary condition that $a_i(r)$ vanish at cutoff surface

$$a_i(r) = \int_{r_c}^r dw \frac{1}{w^3 f(w)} \left(\int_{r_+}^w ds s S_i(s) - 2\sqrt{3}gQ \left(\frac{j_i(w)}{w^4} - \frac{j_i(r_+)}{r_+^4} \right) \right). \quad (\text{B4})$$

After some algebra, eq. (B2) is reduced to

$$j_i''(r) - \frac{3}{r} j_i'(r) - \frac{12Q^2}{r^8 f(r)} j_i(r) = \zeta_i(r), \quad (\text{B5})$$

where

$$\zeta_i(r) = -\frac{12Q^2}{r^4 f(r)} \frac{j_i(r_+)}{r_+^4} + \frac{2r^2}{\sqrt{f(r_c)}} S_{ri}(r) - \frac{2\sqrt{3}Q}{gr^4 f(r) \sqrt{f(r_c)}} \int_{r_+}^r ds s S_i(s). \quad (\text{B6})$$

And then we can write out a particular solution to (B5)

$$j_P(r) = b_1(r)j_{H_1}(r) + b_2(r)j_{H_2}(r), \quad (\text{B7})$$

where

$$b_1(r) = - \int_r^{r_c} dx \frac{j_{H_2}(s)\zeta_i(s)}{s^3}, \quad (\text{B8})$$

$$b_2(r) = r^3 \partial_v \beta_i + \int_r^{r_c} ds \left(\frac{j_{H_1}(s)\zeta_i(s)}{s^3} + 3s^2 \partial_v \beta_i \right). \quad (\text{B9})$$

and

$$j_{H_1}(r) = r^4 f(r), \quad (\text{B10})$$

$$j_{H_2}(r) = r^4 f(r) \int_r^{r_c} \frac{ds}{s^5 f(s)^2}. \quad (\text{B11})$$

are two linearly independent homogeneous solutions of (B5). Here, the $3s^2 \partial_v \beta_i$ term is added to cancel the divergence of the integral. With the above formulas, the general solution to (B5) can be represent as

$$j_i(r) = j_P(r) + C_3 j_{H_1}(r) + C_4 j_{H_2}(r). \quad (\text{B12})$$

In summary,

$$\begin{aligned} j_i(r) = & -r^4 f(r) \int_r^{r_c} ds s f(s) \zeta_i(s) \int_s^{r_c} \frac{dw}{w^5 f(w)^2} + r^4 f(r) \left(\int_r^{r_c} \frac{ds}{s^5 f(s)^2} \right) \\ & \left(r^3 \partial_v \beta_i + \int_r^{r_c} ds (s f(s) \zeta_i(s) + 3s^2 \partial_v \beta_i) \right) + C_3 j_{H_1}(r) + C_4 j_{H_2}(r). \end{aligned} \quad (\text{B13})$$

To obtain $j_i(r_+)$, we take $r \rightarrow r_+$ limit to (B13) and get

$$\frac{j_i(r_+)}{r_+^4} = \frac{r_c^3 \partial_v \beta_i + \int_{r_+}^{r_c} ds s f(s) \zeta_i(s) + C_4}{r_+^5 f'(r_+)} \quad (\text{B14})$$

Noth that, $\zeta_i(r_+)$ also contains $j_i(r_+)$, thus (B14) is in fact the equation related to $j_i(r_+)$. After solving the above equation, we can obtain $j_i(r_+)$

$$\begin{aligned} \frac{j_x(r_+)}{r_+^4} = & \frac{C_4}{r_c^5 f'(r_c)} - \frac{\sqrt{3} Q F_{xv}^{\text{ext}} (-r_+ + r_c)^2}{g r_+ r_c^7 f'(r_c)} \\ & + \frac{\partial_x M \left(-Q^2 (r_+^3 - 3r_+^2 r_c + 2r_c^3) + r_+^2 r_c^2 (r_+^4 r_c - r_+ r_c^4 + 6M(-r_+ + r_c)) \right)}{r_+^3 r_c^{12} f(r_c)^{3/2} f'(r_c)} \\ & + \frac{\partial_v \beta_x \left(-Q^2 (r_+^3 - 3r_+^2 r_c + 2r_c^3) + r_+^2 r_c^2 \left(6M(-r_+ + r_c) + r_+ r_c (r_+^3 + r_c^3 (-1 + \sqrt{f(r_c)})) \right) \right)}{r_+^3 r_c^8 \sqrt{f(r_c)} f'(r_c)} \\ & + \frac{Q \partial_x Q \left(Q^2 (-r_+^3 + r_c^3) - r_c^2 (-2M(5r_+^3 - 6r_+^2 r_c + r_c^3) + r_c (r_+^6 + r_+^3 r_c^3 - 3r_+^2 r_c^4 + r_c^6)) \right)}{r_+^3 r_c^{14} f(r_c)^{3/2} f'(r_c)} \\ & - \kappa_{\text{cs}} \left(\frac{4\sqrt{3} g Q^3 (\partial_z \beta_y - \partial_y \beta_z) (r_+^6 - 3r_+^2 r_c^4 + 2r_c^6)}{r_+^6 r_c^{11} f(r_c) f'(r_c)} + \frac{12 Q^2 F_{zy}^{\text{ext}} (-r_+ + r_c)^2 (r_+ + r_c)^2}{r_+^4 r_c^9 \sqrt{f(r_c)} f'(r_c)} \right) \end{aligned} \quad (\text{B15})$$

In addition, there is a useful equation

$$j'_i(r_c) = -\frac{r_c^2 \partial_v \beta_i}{f(r_c)} + C_3(r^4 f(r))'|_{r=r_c} - \frac{C_4}{r_c f(r_c)}. \quad (\text{B16})$$

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